THE PROBLEM OF ANOMALOUS FILTRATION AND SOLUTE TRANSPORT IN AN INHOMOGENEOUS POROUS MEDIUM

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In this work, filtration and solute transport in a one-dimensional medium of fractal structure is considered.

Let the area of study of the problem consist of $R\{0 \le x < \infty\}$. Initially, the area is filled with a fluid without solute. The process of solute transport, taking into account anomalous effects, can be described by the equation [15]

$$\frac{\partial c}{\partial t} = D\frac{\partial^{\beta} c}{\partial x^{\beta}} - \frac{\partial (vc)}{\partial x}, \tag{1}$$

where c is the concentration of solid particles in the fluid, v is the filtration velocity, D is the diffusion coefficient, β is the order of derivative, t is the time, x is the coordinate.

The anomalous filtration velocity is defined as [3]

$$v = -\frac{k}{\mu}\frac{\partial^{\gamma} p}{\partial x^{\gamma}} \tag{2}$$

where p is the pressure, μ is the viscosity coefficient of the suspension, k is the permeability coefficient, and γ is the order of derivative.

The continuity equation of the flow of a compressible fluid through a porous medium can be written as [23]

$$\frac{\partial (\rho m)}{\partial t} + div(\rho \vec{v}) = 0, \tag{3}$$

where m is the porosity coefficient, ρ is the density of the liquid.

We use the equations of state of an elastic fluid and an elastic porous medium [23]

$$\rho = \rho_0 (1 + \beta_l (p - p_0)), \ m = m_0 + \beta_m (p - p_0), \tag{4}$$

where β_l is the volume compression coefficient of the liquid, β_m is the elasticity coefficient of the medium, ρ_0 is the initial density of the liquid, p_0 is the initial pressure.

Substituting (2), (4) into (3), we can obtain the piezoconductivity equation with a fractional derivative

$$\frac{\partial p}{\partial t} = \chi \frac{\partial^{\gamma+1} p}{\partial x^{\gamma+1}}, \tag{5}$$

where $\chi = k / \mu\beta^*$ is the piezoconductivity coefficient, β^* is the elastic compressibility coefficient of the medium.

So, we obtain a system of suspension filtration and solute transport equations consisting of the balance equation (1), Darcy's law (2) and the piezoconductivity equation (5)

$$\frac{\partial c}{\partial t} = D \frac{\partial^{\beta} c}{\partial x^{\beta}} - \frac{\partial (vc)}{\partial x},$$

$$v = -\frac{k}{\mu} \frac{\partial^{\gamma} p}{\partial x^{\gamma}}, \tag{6}$$

$$\frac{\partial p}{\partial t} = \chi \frac{\partial^{\gamma+1} p}{\partial x^{\gamma+1}}.$$

The initial and boundary conditions of the problem have the following form

$$c(0, x) = 0, \tag{7}$$

$$c(t, 0) = c_0, \ c_0 = \text{const}, \tag{8}$$

$$\frac{\partial c}{\partial x}(t, \infty) = 0, \tag{9}$$

$$p(0, x) = p_0, \quad p_0 = \text{const}, \tag{10}$$

$$p(t,0)=p_c\ ,\ p_c>p_0,\ \ p_c=\text{const}\ , \tag{11}$$

$$\frac{\partial p}{\partial x}(t,\infty)=0. \tag{12}$$

To solve the problem (6) — (12), we use the finite difference method. To accomplish this, we will construct a grid in the area R in the form $\omega_{h\tau}=\left\{\left(t_j,x_i\right),\ t_j=\tau j,\ x_i=ih,\ j=0,1,...,J,\ i=0,1,...,\ \tau=T/J\right\}$, where h is the grid step in the direction of x, τ is the grid step in time, T is the maximum time during which the process is investigated.

Instead of the functions $c(x,t)$, $v(x,t)$ and $p(x,t)$, we will consider net functions, the values of which in the nodes $(x_i,\ t_j)$, respectively, we denote c_i^j , v_i^j and p_i^j .

On the grid $\omega_{h\tau}$, we approximate the first equation of system (6) as follows [12,14,15]

$$\frac{c_i^{j+1}-c_i^j}{\tau}=\frac{D}{\Gamma(3-\beta)h^{\beta}}\sum_{l=0}^{i-1}\left(c_{i-(l-1)}^j-2c_{i-l}^j+c_{i-(l+1)}^j\right)\left((l+1)^{2-\beta}-l^{2-\beta}\right)-$$
$$-\frac{(v)_{i+1}^j c_{i+1}^{j+1}-(v)_{i-1}^j c_{i-1}^{j+1}}{2h}, \tag{13}$$

where $\Gamma()$ is the gamma function.

For the filtration velocity, we use the following scheme

$$(v)_i^j=-\frac{k}{\mu}\frac{p_{i+1}^j-\gamma p_i^j}{\Gamma(2-\gamma)h^{\gamma}} \tag{14}$$

The third equation of system (6) is approximated as

$$\frac{p_i^{j+1}-p_i^j}{\tau}=\frac{\chi}{\Gamma(3-\gamma)h^{\gamma}}\sum_{l=0}^{i-1}\left(p_{i-(l-1)}^j-2p_{i-l}^j+p_{i-(l+1)}^j\right)\left((l+1)^{2-\gamma}-l^{2-\gamma}\right) \tag{15}$$

The initial and boundary conditions are approximated as

$$c_i^j=0,\qquad i=\overline{0,N},\ j=0, \tag{16}$$

$$c_i^j=c_0,\qquad i=0,\ j=\overline{0,J}, \tag{17}$$

$$\frac{c_i^{j+1} - c_i^j}{h} = 0, \qquad i = N, \quad j = \overline{0, J}, \qquad (18)$$

$$p_i^j = p_0 = \text{const}, \qquad i = \overline{0, N}, \quad j = 0, \qquad (19)$$

$$p_i^j = p_c, \qquad i = 0, \; j = \overline{0, J}, \qquad (20)$$

$$\frac{p_i^{j+1} - p_i^j}{h} = 0, \qquad i = N, \quad j = \overline{0, J}, \qquad (21)$$

where N is a sufficiently large number for which equation $c_N^j = 0$ is approximately satisfied.

The sequence of calculations is as follows: first, p_i^{j+1} is determined from the finite difference scheme (15), then the anomalous filtration velocity are calculated from (14), after that c_i^j is determined on the ($j+1$) layer from the difference equations (13). The following values of the initial parameters were used in the calculations: $k = 10^{-13} m^{1+\gamma}$, $\mu = 5 \cdot 10^{-3} Pa \cdot s$, $\beta^* = 3 \cdot 10^{-8} Pa^{-1}$, $p_c = 5 \cdot 10^5 Pa$, $p_0 = 10^5 Pa$, $c_0 = 0{,}01$ and $D = 10^{-5} m^{\beta} / s$.

The problem of filtration and solute transport in a one-dimensional porous medium with a fractal structure is considered. The solute transport in such media is described by an equation with fractional derivatives with respect to the coordinate. The calculation results show that a decrease in the order of derivative in the filtration equation from 1 leads to an increase in pressure and filtration velocity. A decrease in the exponent of the derivative in the diffusion term from 2 leads to “acceleration” of the diffusion process. At the same time, with a decrease in the order of the derivative in the anomalous filtration equation from 1 and the order of the derivative in the diffusion term of the solute transport equation from 2, a wider distribution of concentration profiles can be observed.

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